## SHORT NOTE

# Percentiles of normal hearing-threshold distribution under free-field listening conditions in numerical form 

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(Received 7 February 2005, Accepted for publication 19 April 2005)

Keywords: Hearing threshold, Individual difference, Statistical distribution, Free-field listening
PACS number: $43.66 . \mathrm{Cb}$ [DOI: 10.1250/ast.26.447]

## 1. Introduction

In an earlier work [1], the present authors surveyed studies on normal hearing thresholds for pure tones of frontal incidence under binaural, free-field listening conditions. They showed first that the threshold variation can be approximated by a normal distribution. Then they calculated the standard deviations (SDs) of hearing thresholds as a function of frequency from 25 Hz to 18 kHz , integrating measurement data of 10 studies. The number of listeners involved in those studies was more than 300 for some frequencies.

Using those SDs and the normative threshold values of the ISO standard [2], the normal threshold distribution profile can be determined for nearly the entire range of audible frequencies. Nevertheless, practically speaking, a table of representative values of threshold distributions may be more convenient to use than the mean and SD values: users may want to know, without further calculation, the sound pressure level of a tone that some percentage of the population can perceive.

This Short Note is intended as a supplement to Kurakata et al.'s study, by summarizing representative percentiles of threshold distribution in a tabular form. First, their SDs are reviewed in Sect. 2.1. Then, using these SDs, percentiles of the threshold distribution are presented in Sect. 2.2.

## 2. Construction of a percentile table

### 2.1. Refinement of SDs

The SD curve (Fig. 3 in Ref. [1]) has several peaks and valleys along the frequency axis. Since individual differences in thresholds are not likely to vary greatly between adjacent measurement frequencies, such irregularity could be attributed to the SD estimation procedure: each SD was calculated by integrating measurements of different studies with different numbers of listeners. Therefore, a certain smoothing process is required to obtain a more plausible curve.

[^0]For the purpose of smoothing, a $k$ th-degree polynomial of logarithmic frequency, $\log f$,

$$
\hat{\sigma}(f)=c_{0}+c_{1}(\log f)+c_{2}(\log f)^{2}+\ldots+c_{k}(\log f)^{k}
$$

was fitted to the SD curve, $\sigma(f)$. Table 1 shows the $\mathrm{SDs}, \sigma$ 's, adopted from Ref. [1] for the polynomial approximation. The SDs that were obtained from only one laboratory were not included in this calculation. They may be less reliable than those at other frequencies that were obtained by combining measurements of many laboratories. Furthermore, the SD at 18 kHz was eliminated, although it is included in the table.

Table 1 Standard deviations (SDs) of normal hearingthreshold distribution, in dB .

| Frequency (Hz) | $\hat{\sigma}$ | $\sigma$ | $n$ |
| :---: | :---: | :---: | :---: |
| 31.5 | 7.4 | 7.4 | 47 |
| 40 | 6.5 | 6.5 | 38 |
| 50 | 6.1 | 6.2 | 39 |
| 63 | 5.4 | 5.3 | 118 |
| 80 | 4.6 | 4.4 | 37 |
| 100 | 4.0 | 4.2 | 37 |
| 125 | 3.8 | 4.2 | 179 |
| 160 | 3.8 | 3.2 | 25 |
| 200 | 3.8 | 3.9 | 26 |
| 250 | 3.8 | 4.1 | 174 |
| 315 | 3.8 | 3.6 | 41 |
| 400 | 3.6 | 3.3 | 41 |
| 500 | 3.5 | 4.0 | 172 |
| 630 | 3.6 | 3.7 | 41 |
| 750 | 3.8 | - | - |
| 800 | 3.9 | 4.0 | 41 |
| 1 k | 4.3 | 4.0 | 345 |
| 1.25 k | 4.7 | 4.1 | 106 |
| 1.5 k | 4.9 | - | - |
| 1.6 k | 5.0 | 5.9 | 106 |
| 2 k | 5.1 | 4.7 | 251 |
| 2.5 k | 5.0 | 4.9 | 106 |
| 3 k | 4.9 | 5.4 | 51 |
| 3.15 k | 4.9 | 5.2 | 96 |
| 4 k | 4.9 | 4.1 | 332 |
| 5 k | 5.0 | 4.8 | 128 |
| 6 k | 5.3 | 5.4 | 51 |
| 6.3 k | 5.4 | 5.7 | 118 |
| 8 k | 5.8 | 5.8 | 330 |
| 9 k | 6.0 | 6.1 | 105 |
| 10 k | 6.2 | 6.2 | 241 |
| 11.2 k | 6.9 | 6.8 | 85 |
| 12.5 k | 8.2 | 7.9 | 189 |
| 14 k | 10.9 | 10.2 | 161 |
| 15 k | - | 15.3 | 100 |
| 16 k | 17.5 | 16.8 | 196 |
| 18 k | - | 11.9* | 102 |

[^1]The abrupt change of SD from 17 to 18 kHz that was observed in the original curve may engender an undesirable estimate of fitting.

Akaike's Information Criterion [3,4], or AIC, was used to identify the optimum degree of the equation:

$$
\mathrm{AIC}=-2 M L L+2(k+1)
$$

$M L L$ is the maximum log-likelihood estimated using the equation,

$$
M L L=-(m / 2) \ln \left[\sum_{i=1}^{m}\left\{\sigma\left(f_{i}\right)-\hat{\sigma}\left(f_{i}\right)\right\}^{2} / m\right]
$$

where $m$ represents the number of frequencies involved in the fitting of the equation; for the present calculation, $m=34$ (see Table 1).

Results of this analysis showed that AIC decreased monotonically as the degree of the equation, $k$, increased from 1 to 10; it increased gradually thereafter. Accordingly, the 10th-degree polynomial was chosen for modeling the SD curve. The resulting curve is shown in Fig. 1 with the original SDs for comparison. The SDs obtained by the curve fitting, $\hat{\sigma}$ 's, are presented in Table 1.

### 2.2. Representative percentiles of threshold distribution

Percentiles of the threshold distribution were calculated using the 10 th-degree polynomial obtained in the previous section. ISO standard values [2] were adopted as mean thresholds at frequencies of 31.5 Hz to 16 kHz . The threshold value at 18 kHz was adopted from Ref. [1]. Individual thresholds are expected to distribute around the mean value at each frequency, according to the normal distribution with the $\mathrm{SD}, \hat{\sigma}(f)$.

The $j$ th percentiles of threshold distribution, $P_{j}(f)$, were calculated using the relation,


Fig. 1 Polynomial approximation of the standard deviations (SDs) of threshold distribution.

$$
P_{j}(f)=\operatorname{Th}(f)+q_{j} \hat{\sigma}(f),
$$

where $\operatorname{Th}(f)$ is the normative threshold value of ISO standard and $q_{j}$ is a multiplier transforming the size of SD to a deviation from the mean value of normal distribution that corresponds to the $j$ th percentile: $q_{75} \approx 0.6745$ for $P_{75}$, for example.

Table 2 shows representative percentiles of threshold distribution. Selected percentiles are also represented graphically in Figs. 2(a) and 2(b). Percentiles at 18 kHz were also calculated using the SD in Table 1, although the SD at that frequency was not used in the above-mentioned smoothing process. Referring to the $P_{5}$ values, for example, one can estimate the sound pressure level of a tone that $5 \%$ of young people having a normal hearing ability would be able to perceive. Alternatively, referring to the $P_{25}$ and $P_{75}$ values,


Fig. 2 Graphical representation of the percentiles of threshold distribution chosen from Table 1. (a) From 31.5 Hz to 10 kHz , on the log-frequency scale. (b) From 10 to 18 kHz , on the linear frequency scale.

Table 2 Percentiles of threshold distribution, in dB SPL, around the ISO normative threshold values [2]. The $P_{1}$ and $P_{99}$ values are for informative purposes only.

| Frequency <br> (Hz) | $P_{1}$ | $P_{5}$ | $P_{10}$ | $P_{20}$ | $P_{25}$ | $P_{30}$ | $P_{40}$ | $\begin{gathered} P_{50} \\ \text { (ISO threshold) } \end{gathered}$ | $P_{60}$ | $P_{70}$ | $P_{75}$ | $P_{80}$ | $P_{90}$ | $P_{95}$ | $P_{99}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 31.5 | 42.3 | 47.3 | 50.0 | 53.3 | 54.5 | 55.6 | 57.6 | 59.5 | 61.4 | 63.4 | 64.5 | 65.7 | 69.0 | 71.7 | 76.7 |
| 40 | 36.0 | 40.4 | 42.8 | 45.6 | 46.7 | 47.7 | 49.5 | 51.1 | 52.7 | 54.5 | 55.5 | 56.6 | 59.4 | 61.8 | 66.2 |
| 50 | 29.8 | 33.9 | 36.2 | 38.9 | 39.9 | 40.8 | 42.5 | 44.0 | 45.5 | 47.2 | 48.1 | 49.1 | 51.8 | 54.1 | 58.2 |
| 63 | 24.9 | 28.6 | 30.6 | 33.0 | 33.9 | 34.7 | 36.1 | 37.5 | 38.9 | 40.3 | 41.1 | 42.0 | 44.4 | 46.4 | 50.1 |
| 80 | 20.9 | 24.0 | 25.6 | 27.6 | 28.4 | 29.1 | 30.3 | 31.5 | 32.7 | 33.9 | 34.6 | 35.4 | 37.4 | 39.0 | 42.1 |
| 100 | 17.1 | 19.9 | 21.3 | 23.1 | 23.8 | 24.4 | 25.5 | 26.5 | 27.5 | 28.6 | 29.2 | 29.9 | 31.7 | 33.1 | 35.9 |
| 125 | 13.3 | 15.9 | 17.2 | 18.9 | 19.5 | 20.1 | 21.1 | 22.1 | 23.1 | 24.1 | 24.7 | 25.3 | 27.0 | 28.3 | 30.9 |
| 160 | 9.1 | 11.7 | 13.1 | 14.7 | 15.4 | 15.9 | 16.9 | 17.9 | 18.9 | 19.9 | 20.4 | 21.1 | 22.7 | 24.1 | 26.7 |
| 200 | 5.5 | 8.1 | 9.5 | 11.2 | 11.8 | 12.4 | 13.4 | 14.4 | 15.4 | 16.4 | 17.0 | 17.6 | 19.3 | 20.7 | 23.3 |
| 250 | 2.5 | 5.1 | 6.5 | 8.2 | 8.8 | 9.4 | 10.4 | 11.4 | 12.4 | 13.4 | 14.0 | 14.6 | 16.3 | 17.7 | 20.3 |
| 315 | -0.1 | 2.4 | 3.8 | 5.4 | 6.1 | 6.6 | 7.6 | 8.6 | 9.6 | 10.6 | 11.1 | 11.8 | 13.4 | 14.8 | 17.3 |
| 400 | $-2.2$ | 0.3 | 1.6 | 3.2 | 3.8 | 4.3 | 5.3 | 6.2 | 7.1 | 8.1 | 8.6 | 9.2 | 10.8 | 12.1 | 14.6 |
| 500 | -3.8 | $-1.4$ | -0.1 | 1.4 | 2.0 | 2.5 | 3.5 | 4.4 | 5.3 | 6.3 | 6.8 | 7.4 | 8.9 | 10.2 | 12.6 |
| 630 | $-5.4$ | -3.0 | -1.6 | $-0.1$ | 0.6 | 1.1 | 2.1 | 3.0 | 3.9 | 4.9 | 5.4 | 6.1 | 7.6 | 9.0 | 11.4 |
| 750 | -6.5 | -3.9 | -2.5 | -0.8 | $-0.2$ | 0.4 | 1.4 | 2.4 | 3.4 | 4.4 | 5.0 | 5.6 | 7.3 | 8.7 | 11.3 |
| 800 | -6.9 | -4.2 | -2.8 | -1.1 | $-0.4$ | 0.2 | 1.2 | 2.2 | 3.2 | 4.2 | 4.8 | 5.5 | 7.2 | 8.6 | 11.3 |
| 1 k | $-7.6$ | -4.7 | -3.1 | -1.2 | $-0.5$ | 0.1 | 1.3 | 2.4 | 3.5 | 4.7 | 5.3 | 6.0 | 7.9 | 9.5 | 12.4 |
| 1.25 k | -7.4 | -4.2 | -2.5 | $-0.4$ | 0.3 | 1.0 | 2.3 | 3.5 | 4.7 | 6.0 | 6.7 | 7.4 | 9.5 | 11.2 | 14.4 |
| 1.5 k | $-9.1$ | -5.7 | -3.9 | -1.8 | -0.9 | -0.2 | 1.1 | 2.4 | 3.7 | 5.0 | 5.7 | 6.6 | 8.7 | 10.5 | 13.9 |
| 1.6 k | -9.9 | -6.5 | -4.7 | -2.5 | $-1.7$ | -0.9 | 0.4 | 1.7 | 3.0 | 4.3 | 5.1 | 5.9 | 8.1 | 9.9 | 13.3 |
| 2 k | -13.1 | -9.7 | -7.8 | -5.6 | -4.7 | -4.0 | -2.6 | -1.3 | 0.0 | 1.4 | 2.1 | 3.0 | 5.2 | 7.1 | 10.5 |
| 2.5 k | -15.9 | $-12.5$ | $-10.6$ | -8.4 | -7.6 | -6.8 | $-5.5$ | -4.2 | -2.9 | $-1.6$ | $-0.8$ | 0.0 | 2.2 | 4.1 | 7.5 |
| 3 k | -17.2 | $-13.9$ | $-12.1$ | -9.9 | $-9.1$ | -8.4 | -7.0 | $-5.8$ | -4.6 | -3.2 | $-2.5$ | $-1.7$ | 0.5 | 2.3 | 5.6 |
| 3.15 k | -17.4 | -14.0 | $-12.3$ | $-10.1$ | $-9.3$ | -8.6 | -7.2 | -6.0 | -4.8 | -3.4 | $-2.7$ | $-1.9$ | 0.3 | 2.0 | 5.4 |
| 4 k | -16.7 | -13.4 | $-11.6$ | -9.5 | -8.7 | -7.9 | -6.6 | -5.4 | -4.2 | -2.9 | $-2.1$ | $-1.3$ | 0.8 | 2.6 | 5.9 |
| 5 k | -13.2 | -9.8 | -8.0 | -5.7 | -4.9 | -4.1 | -2.8 | $-1.5$ | -0.2 | 1.1 | 1.9 | 2.7 | 5.0 | 6.8 | 10.2 |
| 6 k | -8.0 | -4.4 | -2.5 | $-0.2$ | 0.7 | 1.5 | 3.0 | 4.3 | 5.6 | 7.1 | 7.9 | 8.8 | 11.1 | 13.0 | 16.6 |
| 6.3 k | $-6.5$ | -2.8 | -0.9 | 1.5 | 2.4 | 3.2 | 4.6 | 6.0 | 7.4 | 8.8 | 9.6 | 10.5 | 12.9 | 14.8 | 18.5 |
| 8 k | -0.8 | 3.1 | 5.2 | 7.8 | 8.7 | 9.6 | 11.1 | 12.6 | 14.1 | 15.6 | 16.5 | 17.4 | 20.0 | 22.1 | 26.0 |
| 9 k | 0.1 | 4.1 | 6.3 | 8.9 | 9.9 | 10.8 | 12.4 | 13.9 | 15.4 | 17.0 | 17.9 | 18.9 | 21.5 | 23.7 | 27.7 |
| 10 k | $-0.6$ | 3.6 | 5.9 | 8.7 | 9.7 | 10.6 | 12.3 | 13.9 | 15.5 | 17.2 | 18.1 | 19.1 | 21.9 | 24.2 | 28.4 |
| 11.2 k | -3.0 | 1.7 | 4.2 | 7.2 | 8.4 | 9.4 | 11.3 | 13.0 | 14.7 | 16.6 | 17.6 | 18.8 | 21.8 | 24.3 | 29.0 |
| 12.5 k | -6.7 | -1.1 | 1.8 | 5.4 | 6.8 | 8.0 | 10.2 | 12.3 | 14.4 | 16.6 | 17.8 | 19.2 | 22.8 | 25.7 | 31.3 |
| 14 k | $-7.0$ | 0.4 | 4.4 | 9.2 | 11.0 | 12.7 | 15.6 | 18.4 | 21.2 | 24.1 | 25.8 | 27.6 | 32.4 | 36.4 | 43.8 |
| 16 k | $-0.5$ | 11.4 | 17.8 | 25.5 | 28.4 | 31.0 | 35.8 | 40.2 | 44.6 | 49.4 | 52.0 | 54.9 | 62.6 | 69.0 | 80.9 |
| 18 k | 41.3 | 49.5 | 53.8 | 59.1 | 61.1 | 62.8 | 66.1 | 69.1* | 72.1 | 75.4 | 77.1 | 79.1 | 84.4 | 88.7 | 96.9 |

* adopted from Ref. [1]
one can estimate the sound level range that would include thresholds of $50 \%$ of young people.

Note that the $P_{1}$ and $P_{99}$ values are susceptible to errors in SD estimation since they are located on the tails of the threshold distribution. For this reason, they might be less reliable than the other percentiles in Table 2 and are to be used for informative purposes only. In particular, the $P_{1}$ values might have been estimated as slightly lower because our hearing ability must have a certain limit in improvement; consequently, the threshold variation might deviate from the normal distribution at the lower tail.

## Acknowledgment

The authors are grateful to an anonymous reviewer for
helpful suggestions and critical reading of the manuscript.

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[^1]:    $\hat{\sigma}$ : SDs obtained by polynomial fitting
    $\sigma$ : original SDs adopted from Ref. [1]
    $n$ : number of listeners whose data were used in the SD calculation [1]

    * not used in polynomial fitting

